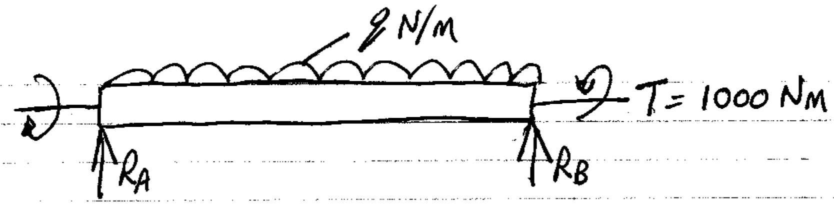
Q1.



$L = 2m$
$d_o = 75mm$
$d_i = 60mm$

$A = \frac{\pi}{4}\left[d_o^2 - d_i^2\right] = \underline{1590\ mm^2}$

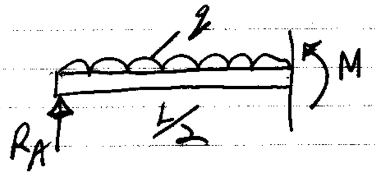
$J = \frac{\pi}{32}\left[d_o^4 - d_i^4\right] = \underline{1.834 \cdot 10^6\ mm^4}$

$I = \frac{J}{2} = \underline{0.917 \cdot 10^6\ mm^4}$

$\text{Mass} = \rho A L = 7800 \cdot 1590 \cdot 10^{-6} \cdot 2 = \underline{\underline{24.8}}\ kg$

$\therefore\ \text{UDL}\ q = \frac{24.8 \cdot 9.81}{2} = \underline{121.6}\ N/m$

$R_A = R_B = \frac{qL}{2} = \underline{121.6\ N}$ [12 marks]



$M - R_A \cdot \frac{L}{2} + q\frac{L}{2} \cdot \frac{L}{4} = 0$

$M = R_A\frac{L}{2} - \frac{qL^2}{8} = 121.6 - 60.8$

$= \underline{\underline{60.8}}\ Nm$ [4 marks]

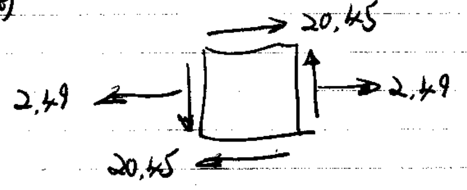
Bending Stress $\sigma = \frac{My}{I} = 60.8 \cdot \frac{75 \cdot 10^{-3}}{2} \cdot \frac{1}{0.917 \cdot 10^{-6}}$

$= \underline{\underline{2.49}}\ MPa$ [4 marks]

Torsional Shear Stress $\tau = \frac{Tr}{J} = 1000 \cdot \frac{75 \cdot 10^{-3}}{2} \cdot \frac{1}{1.834 \cdot 10^{-6}}$

$= \underline{\underline{20.45}}\ MPa$ [4 marks]

Q1 (cont)



Mohr's Circle

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{2.49}{2}$$
$$= \underline{1.245\ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{2.49}{2}\right)^2 + 20.45^2}$$
$$= \underline{20.49\ MPa}$$

Principal Stresses $\sigma_1 = C + R = \underline{21.74\ MPa}$

$\sigma_2 = C - R = \underline{-19.25\ MPa}$

Max Shear Stress $\tau_{max} = R = \underline{20.49\ MPa}$

[9 marks]

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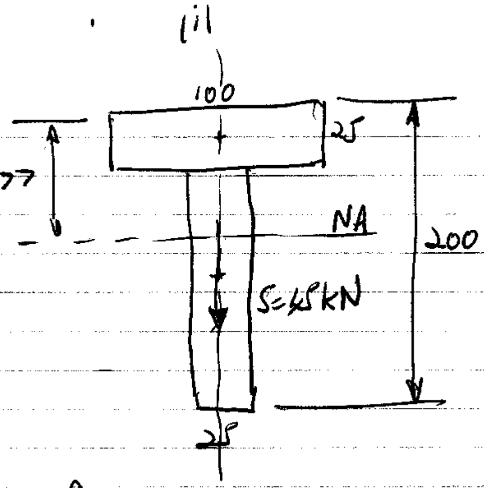
Q2. (i)



(a) <u>2nd Moment of Area</u>

$$I_{NA} = \frac{bd^3}{12} + Ah^2 = \frac{100.25^3}{12} + 100.25.(77-12.5)^2$$

$$+ \frac{25.175^3}{12} + 25.175.(112.5-77)^2$$

$$= 130{,}208 + 10.4.10^6$$

$$+ 11.165.10^6 + 5.514.10^6$$

$$= \underline{2.721.10^7 \text{ mm}^4}$$

[10 marks]

(b) <u>Flange-web join</u>

$$\tau = \frac{SA\bar{y}}{Iz} = \frac{45.10^3.100.25.[77-12.5]}{2.72.10^7.100}$$

$$= \underline{2.67 \text{ MPa}} \text{ above join}$$

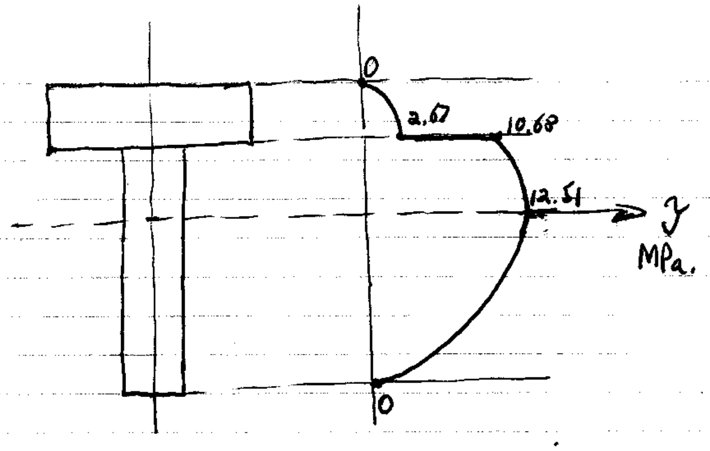
$$\tau = 2.67.\frac{100}{25} = \underline{10.68 \text{ MPa}} \text{ below join}$$

<u>Neutral Axis</u>

$$\tau = \frac{SA\bar{y}}{Iz} = \frac{45.10^3.123.25.61.5}{2.72.10^7.25} = \underline{12.51 \text{ MPa}}$$

[18 marks]

Q2 (cont'd) Sketch



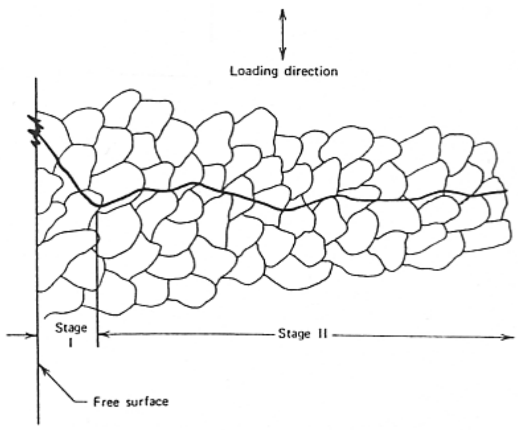
[5 marks]

Q3 (a) Crack initiation, crack propagation and final fracture, as follows:

(i) Stage I crack growth: development of persistent slip bands resulting from dislocations moving along crystallographic planes leading to stress concentrations

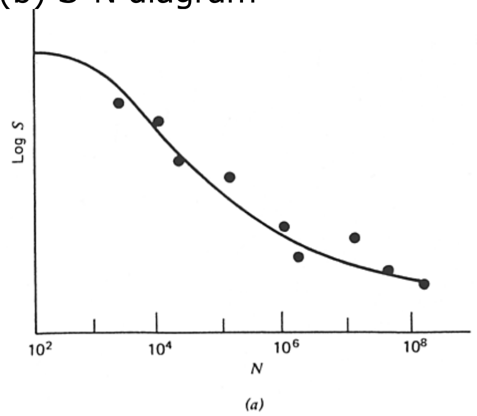
(ii) Stage II crack growth: the fatigue cracks tend to coalesce and grow along planes of maximum tensile stress range.

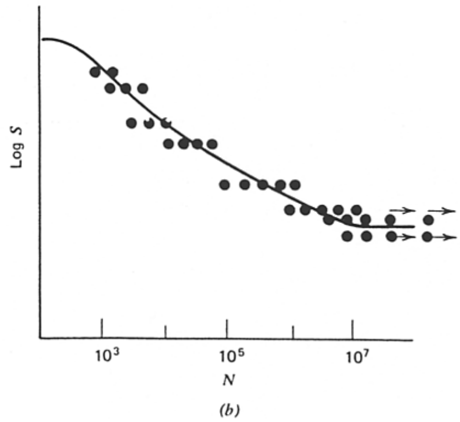
(iii) Final fracture; this occurs when the crack reaches a critical length and results in either ductile tearing (plastic collapse) or cleavage (brittle fracture)



[10 marks]

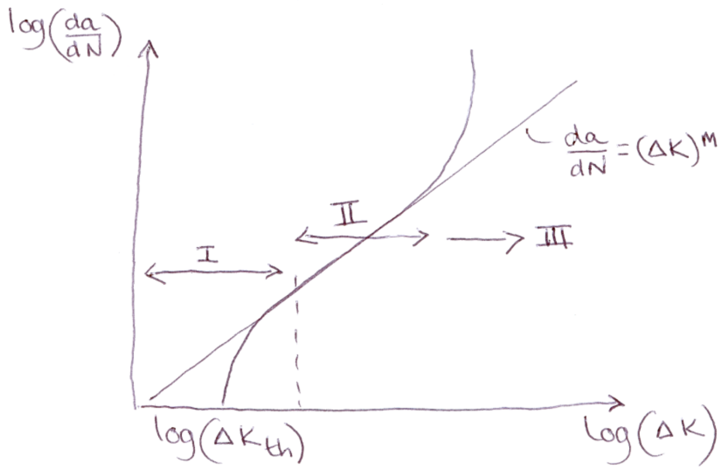
(b) S-N diagram





Paris law shows there is a logarithmic linear relationship between crack growth rate and the stress intensity factor range during cyclic loading of cracked components

$$\frac{da}{dN} = C\,(\Delta K)^m$$



[10 marks]

(c)

At fracture

K_{IC} = 1.12 $\sigma(\pi\, a_{crit})^{1/2}$

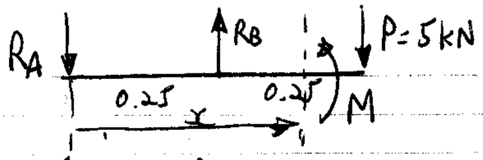
a_{crit} = critical crack length

Therefore
$a_{crit} = (1/\pi) \mathrm{x} (K_{IC} / 1.12\sigma)^2$

$a_{crit} = (1/\pi) \mathrm{x} (200/ 1.12.150)^2$ = 0.451m

[13 marks]

Q4. (a)



$\downarrow \quad R_A + 5000 = R_B$

$EI = 10^4 \; Nm^2$

$\circlearrowright B \quad R_A .0.25 = 5000.0.25.$

$\underline{R_A = 5{,}000 \text{ N}}$

$R_B = \underline{10{,}000 \text{N}}$

[4 marks]

<u>Moments</u> $\quad M + R_A x - R_B \langle x - 0.25 \rangle = 0$

$$EI \frac{d^2y}{dx^2} = -M = R_A x - R_B \langle x - 0.25 \rangle$$

$$EI \frac{dy}{dx} = R_A \frac{x^2}{2} - R_B \frac{\langle x - 0.25 \rangle^2}{2} + A$$

$$EIy = \frac{R_A x^3}{6} - R_B \frac{\langle x - 0.25 \rangle^3}{6} + Ax + B.$$

[8 marks]

<u>Boundary Conditions</u>

when $x = 0 \; y = 0 \quad \therefore \quad B = 0$

when $x = 0.25 \; y = 0 \quad \therefore \quad 0 = \frac{R_A (0.25)^3}{6} + A.0.25$

$$\therefore \quad A = -\frac{R_A (0.25)^2}{6} = \underline{\underline{-52.08}}$$ [4 marks]

(i) when $x = 0.5 \quad EIy = \frac{5000(0.5)^3}{6} - \frac{10{,}000(0.25)^3}{6} - 52.08.0.5$

$$= 104.2 - 26.04 - 26.04$$
$$= \underline{52.12}$$

$\therefore y = 52.12.10^{-4} = \underline{\underline{5.21 \text{ mm}}}$ [4 marks]

Q4(a)(cont)

(ii) when $x = 0.5$ $EI\frac{dy}{dx} = 5000.\frac{0.5^2}{2} - 10{,}000\frac{(0.25)^2}{2} - 52.08$

$= 625 - 312.5 - 52.08$

$= \underline{260.42}$

$\frac{dy}{dx} = 260.42.10^{-4} = 0.026$ radians

$= \underline{\underline{1.5^\circ}}$ [4 marks]

(b) At B $R_B = 10{,}000$ N

$k = 20.10^6$ N/m

$\therefore \delta_B = \frac{10{,}000}{20.10^6} = \underline{\underline{0.5\text{ mm}}}$

Boundary condition

when $x = 0.25$ $y = 0.5$

$\therefore 0.5.10^{-3}.EI = \frac{5000.0.25^3}{6} + \cancel{10{,}000} A.0.25$

$\therefore A.0.25 = 0.5.10^{-3}.10^4 - \frac{5000.0.25^3}{6}$

$= 5 - 13.02 = \underline{-8.02}$

$\therefore A = \underline{\underline{-32.08}}$

$\therefore$ when $x = 0.5$ $EIy = \frac{5000.(0.5)^3}{6} - \frac{10{,}000.(0.25)^3}{6} - 32.08.0.5$

$EIy = 104.17 - 26.04 - 16.04$

$= \underline{62.09}$

$\therefore y = 62.09.10^{-4} = \underline{\underline{6.21\text{ mm}}}$

[9 marks]

(5) Hoop & axial stresses due to pressure of p:

$$\sigma_\theta = \frac{pR}{t} = 40p$$

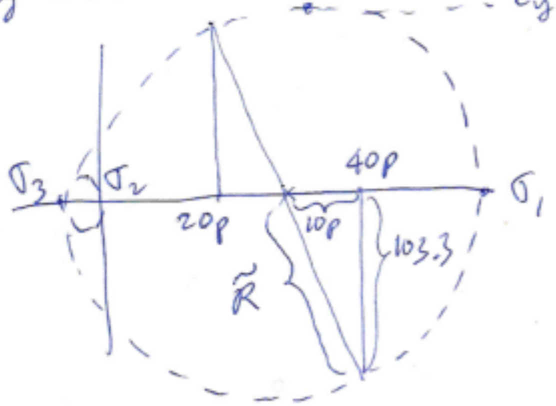
$$\sigma_z = \frac{pR}{2t} = 20p$$

shear stress due to the torque of 1 kNm:

$$\tau = \frac{TR}{I_p} \quad \text{where} \quad I_p = \frac{\pi}{32}(D_o^2 - D_i^2) = \frac{\pi}{32}(80^2 - 78^2) = 387\,294 \text{ mm}^4$$

$$= \frac{1\text{kNm} \times 40\text{ mm}}{387\,294 \text{ mm}^4} = 103.3 \text{ MPa}$$

(i) Using Tresca criterion: $\tilde{\tau}_y = 125$ MPa



$$\tilde{R} = \sqrt{(10p)^2 + (103.3)^2} \le \tau_y$$

$$100p^2 + 103.3^2 \le 125^2$$

$$p^2 \le \frac{125^2 - 103.3^2}{100}$$

$$\therefore p_{max} = \sqrt{\frac{125^2 - 103.3^2}{100}} \text{ MPa} = \underline{\underline{7 \text{ MPa}}}$$

(ii) Using von Misses criterion:

The principle stresses are: ; where $\tilde{R} = \sqrt{(10p)^2 + (103.3)^2}$

$\sigma_1 = 30p + \tilde{R}$

$\sigma_2 = 0$

$\sigma_3 = 30p - \tilde{R}$

$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \leq 2\sigma_y^2$

$(30p + \tilde{R})^2 + (30p - \tilde{R})^2 + \tilde{R}^2 \leq 2\sigma_y^2$

$2(900p^2 + \tilde{R}^2) + \tilde{R}^2 \leq 2\sigma_y^2$

$1800p^2 + 3\tilde{R}^2 \leq 2\sigma_y^2$

$1800p^2 + 3(100p^2 + 103.3^2) \leq 2 \times 250^2$

$2100p^2 + 3 \times 103.3^2 \leq 2 \times 250^2$

$p^2 \leq \dfrac{2 \times 250^2 - 3 \times 103.3^2}{2100}$

$\therefore p_{max} = \underline{\underline{6.65}}$ MPa